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A Dynamic Look at Subprime Loan Performance

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Mortgage performance is typically studied in terms of the probability or frequency of default and prepayment, a static characterization that does not consider the behavior of a loan before it terminates. Before termination, a loan can be either current or delinquent. A delinquency can last for only a short period of time or for a very long time.

Understanding the dynamic link between delinquency and loan termination is important for several reasons. For example, the delinquency behavior of loans can impact the payment streams of securities with underlying mortgage collateral. In addition, regulators, lenders, and other secondary market participants can benefit from understanding the risk of termination associated with delinquent mortgages.

High-risk subprime mortgages provide an ideal laboratory for studying the dynamic nature of mortgage performance because these loans tend to default and terminate at high rates (see Alexander et al. [2002], Pennington-Cross [2003], Cowan and Cowan [2004], and Capozza and Thomson [2005]). Subprime lending tends to be

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concentrated in low-income and minority areas and in areas with troubled economic conditions. Subprime borrowers also tend to have poor credit characteristics, be less knowledgeable about the mortgage process, and be less satisfied with their mortgages. These are characteristics generally found to be consistent with trouble in meeting financial commitments (Pennington-Cross [2002], Calem, Cillen, and Wachter [2004], and Courchane, Surette, and Zorn [2004]).

We examine the implications of delinquency for the performance of subprime mortgages. That is, does delinquency have any predictive power for the future performance of a mortgage? In addition, while it seems intuitively obvious that delinquency naturally leads to default, we also examine whether delinquency increases or reduces the probability that a loan will terminate through prepayment. We find evidence suggesting that when a loan is delinquent over a long period of time, prepayments dominate defaults as the primary termination.

I. Motivation and Literature Review

We examine the history of a loan until it defaults, which we define as entering foreclosure proceedings or becoming real estate owned by the lender, or until the loan is terminated through prepayment. Exhibit 1 provides a conceptual overview of the dynamic relationship between delinquency and the final outcome or termination of the loan.[1]

In each month that a loan is alive or still active, it can be either current or delinquent. Loans can terminate at any time, but can default only after being delinquent; yet delinquency can lead to any other state (current, default, or prepayment). In addition, prepaid loans can be delinquent or current in the previous month.[2]

Delinquency does play an important part in the path a loan takes to termination. Since a loan must necessarily be delinquent before default, it may seem obvious that delinquent loans must be more likely to default. Mitigating factors can retard the transition from delinquency to default, though, the most important being prepayment of the mortgage.

A rational borrower may attempt to avoid the costs of foreclosure, which can be substantial and include legal fees besides a negative credit report. Negative credit reports can impact the cost of credit in the future. One way to avoid these costs is to sell the property and thus prepay the mortgage. Lenders too have incentives to avoid foreclosure costs through workout arrangements with delinquent borrowers. Many of these workouts, such as short refinances, result in prepayment of the mortgage.[3]

An important element to consider is that default and prepayment are competing risks. Increases in the probability of prepayment must necessarily lead to reduction in either the probability of continuing the mortgage or the probability of default.

The economic motives behind prepayments in the case of a seriously delinquent mortgage are distinct from the traditional motives tor prepayment. Customary drivers of prepayments include drops in interest rates and trigger events such as job loss or divorce. Prepayments of delinquent mortgages, however, can be viewed as distressed prepayments brought about by borrower or lender desire to avoid a default.

The current equity status of the property is a key determinant of whether a delinquent mortgage will prepay or will default. From the borrower's perspective, a positive equity position makes the borrower more likely to attempt to preserve such a position by selling rather than letting the property go into foreclosure. From the lenders perspective, the opposite is true in the case of a property with positive equity. If the borrower does not want to sell the house, the least costly alternative may be to foreclose, sell, and use the proceeds to satisfy the debt. The net impact of current equity on defaults and prepayments is thus an open empirical question.

There is no reason to assume that the relationship between delinquency and default is linear. For example, Ambrose, Buttimer, and Capone [1997] identify three benefits to delinquency, namely, free rent, income smoothing, and time to cure or the value of delay.

Free rent is received during delinquency because the mortgage is not being paid in a timely Hishion. Borrowers can also not pay their mortgages in an attempt to maintain a standard of living beyond current income streams. This may make most sense for those with highly variable income sources or anticipated permanent increases in income in the near future. Lastly, delinquency by its nature entails a period of delay, and delaying can be valuable because it can buy time to solve the problem. House prices may rise dramatically or the borrower may solve the liquidity problem through a change in job status, seasonal income streams, or improved credit availability. Kau and Kim [1994] discuss the value of delay and the role of house price volatility in an options theory framework.

Borrowers face significant costs while being delinquent. Late fees accrue over time, making it cost more in the long run to cure the loan. In addition, the delinquency is reported to credit agencies, which can have long-term and dramatic impacts on a household. The cost of credit will increase; the availability of credit will lessen; and new positions may be threatened due to credit and background checks. There are significant costs to default that could make prepayment a more attractive option.

Given that delinquency can precede almost any outcome, it is an empirical question as to whether it leads to more defaults, prepayments, or just more delinquency. To examine the influence of delinquency on the future performance of a mortgage, we need to understand the forces that influence the probability a loan will be delinquent and the intensity of the delinquency. Empirical research over the last 30 years has addressed many of the same drivers.

For example, von Furstenberg and Green [1974] and Morton [1975] find that the loan to value (LTV) ratio at origination and the income of the borrower play important roles in mortgage delinquency. Getter [2003] complements these findings by using the 1998 Survey of Consumer Finances to show that borrowers use other non-housing financial assets to help make payments during unexpected periods of financial stress. Chinloy [1995] finds in the United Kingdom during the period 1983 through 1992 that LTV and income are the primary covariates associated with delinquency. Other researchers also find that credit scores, contemporaneous economic conditions, and the incentive structure of the lender all can impact delinquency (Baku and Smith [1998), Caleni and Wachter [1999], Ambrose and Capone [2000].[4]

Ambrose and Capone [1996, 2000] have shown empirically that the behavior of a loan in the past can help to predict its behavior in the future. They find that the length of the first serious delinquency (defined as time spent 90 or more days delinquent) reduces the probability of a second period of serious delinquency (90 days- plus delinquent). In addition, if the loan enters serious delinquency tor a second time, it is less likely to be reinstated. These results provide empirical evidence that die current status of a mortgage is not independent of its status in previous months.

We extend this literature by jointly estimating the probability of being delinquent with the intensity of delinquency measured by the cumulative delinquency rate. We also estimate the impact of the predicted probability' and predicted intensity of delinquency on the probability of default and prepayment in the second step of the estimation. This approach lets us observe and test for the dynamic and non-linear nature of mortgage behavior.[5]

II. Econometric Model

A mortgage's status is the result of joint decisions by the borrower and the lender. The current status—prepaid, defaulted, or continuing—is influenced by the cumulative payment history. Because a mortgage's current outcome is not independent of the previous monthly outcomes, we use a Heckman two-step procedure to control for the endogeneity. We specifically focus on the impact of past delinquency on the current outcome.

In the first step, we estimate the intensity of delinquency, defined as the fraction *oi* the observed life of the loan that it is delinquent. In the second step, we estimate a seemingly unrelated bivariate probit model of mortgage outcomes and include the predicted intensity of delinquency and predicted delinquency probability from the first step.

In the first step of our model, we estimate a double-hurdle Tobit model (Cragg's model) of the intensity' of delinquency because the majority of mortgages have zero incidence of delinquency. The double-hurdle Tobit model separately models the probability of experiencing a delinquency and the intensity. Specifically, let the first hurdle be represented as

$$d_i^* = z_i\alpha + \varepsilon_i \quad (1)$$

where d_i^* is an unobserved measure of the propensity of a mortgage i to be delinquent, z_i is a vector of borrower and loan characteristics, α is a vector of parameters to be estimated, and $\varepsilon_i \sim N(0,1)$. Define a dummy variable, d_i, as:

$$d_i = 1 \text{ if } d_i^* > 0$$
$$d_i = 0 \text{ if } d_i^* \le 0 \quad (2)$$

The second hurdle is given by

$$y_i = \max(x_i\beta + u_i, 0) \quad (3)$$

where y_i is the fraction of the observed life of mortgage i that is delinquent or the intensity of delinquency, x_i is a vector of borrower and loan characteristics, β is a vector of parameters to be estimated, and $u_i \sim N(0,\sigma^2)$.

It is important to note that ε and u are assumed independent. By this we mean that unobserved factors that cause a mortgage to be potentially delinquent are uncorrelated with the unobserved factors that determine the fraction of the observed life that the mortgage is actually delinquent.

The log-likelihood function is given by:

$$L_1 = \sum_0 \ln\left[1 - \Phi(z_i\alpha)\Phi\left(\frac{x_i\beta}{\sigma}\right)\right] + \sum_+ \ln\left[\Phi(z_i\alpha)\frac{1}{\sigma}\phi\left(\frac{y_i - x_i\beta}{\sigma}\right)\right] \quad (4)$$

where Σ_0 denotes the summation over observations with zero delinquency, Φ denotes the standard normal distribution function, Σ_+ denotes the summation over observations with a positive delinquency rate, and Φ denotes the standard normal density function. The log-likelihood function is maximized by choosing the unknown parameters α, β, and σ.

The predicted value of intensity can be calculated using the estimated parameters $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\sigma}$. The predicted value is given by:

$$y_i^* = \begin{cases} \hat{y}_i \text{ if } \hat{y}_i > 0 \\ 0 \text{ if } \hat{y}_i \le 0 \end{cases} \quad (5)$$

where

$$\hat{y}_i = \Phi(z_i\hat{\alpha})\Phi\left(\frac{x_i\hat{\beta}}{\hat{\sigma}}\right) * \left[x_i\hat{\beta} + \frac{\hat{\sigma}\phi\left(\frac{x_i\hat{\beta}}{\hat{\sigma}}\right)}{1-\Phi\left(\frac{x_i\hat{\beta}}{\hat{\sigma}}\right)}\right] \quad (6)$$

Intuitively, $\hat{\gamma}$ equals the probability of delinquency multiplied by the expected value of the delinquency ratio, conditional on a delinquency rate greater than zero.

The second stage of the estimation uses the predicted value of the intensity of delinquency in a seemingly unrelated bivariate probit model of the mortgage outcome. Specifically, we jointly model the probability of default and the probability of prepayment of a mortgage.[6]

The model specification is given by

$$\begin{aligned} \pi_i^{*d} &= w_i^d \delta^d + \varepsilon_i^d \qquad \pi_i^d = 1 \text{ if } \pi_i^{*d} > 0, \text{and 0 otherwise} \\ \pi_i^{*p} &= w_i^p \delta^p + \varepsilon_i^p \qquad \pi_i^p = 1 \text{ if } \pi_i^{*p} > 0, \text{and 0 otherwise} \end{aligned} \tag{7}$$

and

$$\begin{aligned} E[\varepsilon_i^d] &= E[\varepsilon_i^p] = 0 \\ \mathrm{Var}[\varepsilon_i^d] &= \mathrm{Var}[\varepsilon_i^p] = 0 \\ \mathrm{Cov}[\varepsilon_i^d, \varepsilon_i^p] &= \rho \end{aligned} \tag{8}$$

Equation (7) models the probability of default and prepayment of mortgage i (π_i^{*d} and π_i^{*p}, respectively) as a function of loan and borrower characteristics, w_i, including the predicted intensity of delinquency, and unknown parameters δ. The error terms ε_i have a correlation coefficient equal to ρ.

The log-likelihood function for the seemingly unrelated bivariate probit is given by:

$$L_2 = \sum_i \ln \Phi_2\left[(2\pi_i^d - 1)\mathrm{w}_i^d \delta^d, (2\pi_i^p - 1)\mathrm{w}_i^p \delta^p, \rho\right] \tag{9}$$

where Φ_2 denotes the standard bivariate normal cumulative density function. The function is maximized by choosing the parameters δ^d, δ^d, and ρ.[7]

Following Murphy and Topel [1985], we correct the variance-covariance matrix of the bivariate probit model to account for the inclusion of estimated variables as regressors. We follow a procedure outlined in Hardin [2002] to accomplish the correction using the statistical package STATA. The standard errors exhibit very little change as a result of the correction.[8]

III. Data

We draw our sample of loans for the estimation from a dataset consisting of the performance history of the underlying collateral of pools of private-label subprime securitizations available from Loanperformance (LP). Only 30-year fixed-rate loans for home purchase in metropolitan areas are included. The LP database provides information on the loan at origination, including property location, LTV, credit score (FICO), and documentation and prepayment penalty status. The database also includes pool-level information on the provider of the data to LP, as well as monthly information on the age and the status of the loan (current, defaulted, prepaid, or delinquent).

A cross-section of 22,799 loans from January 1996 through May 2003 is selected from the LP database. For each loan, we randomly pick one month in the performance history and compute the intensity of delinquency to that point. This is the fraction of the observed life of the loan that is delinquent. For example, 0 indicates that the loan has never been delinquent, 0.5 indicates that the loan has been delinquent one-half of the time, and 1 indicates that the loan has always been delinquent.

External data from a number of sources are matched to the sample. We use the metropolitan area repeat sales House Price Index from the Office *of* Federal Housing Enterprise Oversight and the balance of the loan to calculate a current loan-to-value ratio. We match the contemporaneous metropolitan area unemployment rate from the Bureau of Labor Statistics to the loan. We also compute the change in the prevailing prime interest rate from the date of loan origination to the current date using Freddie Mac's Primary Mortgage Market Survey as a measure of the change in interest rates affecting the refinancing incentive.

A more detailed description of the variables used in the estimation is in Exhibit 2. Exhibit 3 provides summary statistics for the data.

Identification is achieved in the model using a theory-based specification approach. The double-hurdle model and the bivariate probit model include a common set of covariates such as age of the

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loan and FICO that are chosen on the basis of their theoretical relationship.

One variable, a low documentation binary, is included in the double-hurdle model of cumulative delinquency but not in the bivariate model of default and pre-payment. Low documentation loans are typically used by borrowers with lumpy income streams such as small business owners. Because of the uneven income streams of these borrowers, we would expect to see higher rates of missed payments, but we would not expect to see differing levels of loan termination based on uneven income streams.

Two variables, the change in interest rates and a prepayment penalty binary, are included in the bivariate probit model only.[9] Interest rate changes are theorized to affect prepayments through the refinance incentive and to affect defaults through the option theory of mortgages.[9]

IV. Results

Exhibit 4 presents the results of the first step of the estimation, the double-hurdle Tobit model. The first column reports the results from estimation of the first hurdle [the α vector in Equation (1)], the probability of delinquency, and the second column reports the results from estimation of the second hurdle [the β vector in Equation (3)], the intensity of delinquency. Exhibit 5 reports the results of the second step of the estimation, the seemingly unrelated bivariate probit model [the δ^d and δ^p vectors in Equation (7)].

Because many of the independent variables enter into both the first and the second stages of the estimation, interpretation of the coefficients is not straightforward. For instance, FICO affects the predicted cumulative delinquency frequency by affecting the probability of delinquency as well the level of delinquency, conditional on being delinquent. The predicted intensity of delinquency and the predicted probability of delinquency then affect the probability of default and the probability of prepayment in the seemingly unrelated bivariate probit model.

In the second step, then, FICO has an indirect effect on the probability of default and prepayment through its impact on predicted delinquency probability- and intensity of delinquency, and a direct effect through inclusion of a FICO variable.

Exhibit 6 graphs this relationship and the way FICO ultimately affects default and prepayment probabilities. To interpret the coefficients, we graph the estimated probability of default and prepayment over the range of observed values for each of the continuous independent variables, holding all other variables at their means. For the discrete independent variables, we calculate the percentage change in the estimated probabilities as the variable moves from 0 to 1.

The past delinquency behavior of a loan is strongly positively related to the probability of default and prepayment, as shown in Exhibit 7. This is the direct effect *oi* the intensity' of delinquency, and does not incorporate the indirect effects of variables that caused the delinquency to change in the first place.

As one would expect, as a loan increases in the intensity of delinquency, there is a higher probability that the loan defaults. There is a peak in defaults at 6.3% when the intensity is 0.72 and a slight decline thereafter.

Somewhat surprising is the strength of the impact of past delinquency behavior on prepayments. At an intensity of delinquency of 0.72, the probability of prepayment is 26.3%. This is a strong indicator of distressed prepayments.

One important finding is that delinquency in the subprime market tends to lead to prepayments more than defaults. Prepayments increase more quickly than defaults as the intensity of delinquency increases. The odds ratios for default and prepayment are 3.82 for default and 5.89 for prepayment as the intensity of delinquency increases from 2% to 72%.

As a result, while prepayments are almost always more likely, they are even more prevalent when a loan has been delinquent most of its observed life. Prepayments are 2.93 times more likely when we should see very few defaults (intensity of delinquency = 0.02), and prepayments are 4.16 times more likely when distressed prepayments are very likely (intensity of delinquency = 0.72). These results provide evidence that distressed prepayments rise rapidly, and even more than defaults, in response to extended periods of delinquency.

Exhibits 8A and 8B reflect the marginal effects of LTV at origination and current LTV on our first- and second-stage estimates. The two graphs are practically mirror images. While the origination LTV results reflect the impact of subprime underwriting requirements that higher LTV loans must have compensating factors, the marginal effects of current LTV support the *ruthless default theory* of borrower behavior.

As current LTV crosses the threshold of 100, the probability of default increases exponentially. At an LTV of 100, the probability' of default is 6.8%, rising to 25.9% as LTV climbs to 120. When current LTV is in excess of 100, the value of the property is less than the mortgage outstanding, leading to a ruthless default on the mortgage in an option-theoretic framework.[10]

We also find that prepayments are negatively related to the current LTV. This is consistent with the limited options of a borrower in a severe negative equity option.

Further evidence of distressed prepayments appears in Exhibits 9A and 9B. Delinquent borrowers with positive equity in their property, evidenced by low current LTV, prepay with greater probability than delinquent borrowers without equity. This appears to be a rational response for borrowers who are weighing selling their property and preserving equity compared to borrowers without equity to protect. Delinquent borrowers with positive equity rarely default, while delinquent borrowers without equity default at much higher probabilities. This suggests that, although lenders have incentives to foreclose on properties with positive equity, borrowers are prepaying in advance of that possibility.[11]

Credit scores play an important role in determining the probabilities of prepayment and default both directly and indirectly. Exhibit 10 shows the effects of FICO on the probability of delinquency and the intensity of delinquency Borrowers with low credit scores are delinquent with a 25% probability, and these loans are predicted to be delinquent nearly 20% of their lifetime. Borrowers with credit scores of 750, however, are delinquent with a 3% probability, and their loans will spend just 0.65% of their lives in delinquency.

The combined indirect and direct impact of FICO on default and prepayment is shown in Exhibit 11. At levels of FICO below 570, the probability of default is greater than the probability of prepayment. As expected, defaults decline with FICO, indicating that performance with regard to past financial obligations is a good predictor of current performance. We also find that prepayments increase with credit score. This may be an indication that borrowers with high credit scores are able to cure into prime mortgages.

Exhibit 1 2 reflects the percentage change in our four estimates of interest as each of the continuous independent variables are increased by one standard deviation, holding all other variables at their means. The impacts on the probability' of delinquency, the intensity of delinquency, the probability of default, and the probability of prepayment are shown.

Rising credit scores reduce the probability of delinquency and the intensity of delinquency. An increase in FICO by one standard deviation cuts the probability of default by nearly one-half, while the probability of prepayment increases by nearly one-quarter.

As would be expected, the probability of prepayment is negatively related to changes in interest rates over the life of the loan. Exhibit 13 reports this evidence. Prepayment and (to a lesser extent) default probabilities decline as interest rates rise. This is consistent with the refinancing incentive for prepayment.

The area unemployment rate, included as a proxy for trigger events, has very little impact on our estimated variables. Rising unemployment rates would be theorized to increase delinquency and default probabilities as they potentially increase the financial distress of these borrowers, but we do not find this relationship using the previous month's metropolitan area unemployment rate as an indication of trigger events.

Exhibit 14 shows the percentage change in the discrete independent variables as the variable switches from 0 to 1. The first row reflects the impact of low documentation (LD) on a loan's performance. Low doc increases the probability of delinquency and the intensity of delinquency, but slightly reduces the probabilities of default and prepayment.

The second row shows the impact of prepayment penalties. The presence of a prepayment penalty reduces the probability of prepayment by one-half.

The next series of variables in Exhibit 14 represent the fixed effects of *MIC_group.* MIC_group is a variable in the pool-level Loanperformance data indicating the source of the data (the data provider). Data providers include lenders and servicers in the subprime market. The coefficients can therefore reflect many different sources of heterogeneity in the subprime market derived from origination, underwriting of the pools of loans, owners of the securities, and servicing.

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The results are significant and substantial in all the estimates. Tests of interaction of the MIC_group with delinquency and credit scores proved to be untruthful.

V. Conclusion

The emergence of subprime lending has created many challenges in the marketplace. With the high, and sometimes unexpectedly high, termination rates of subprime loans, one challenge is to come to a more complete understanding of how mortgages terminate. For example, are there paths to termination that indicate whether a loan will ultimately default or prepay?

The evidence is that the long-run delinquency of a loan leads to elevated probabilities of prepayment and default, with a more pronounced response in terms of prepayment. These prepayments are made when a loan is delinquent and are independent of interest rates; as a consequence, we interpret these types of prepayments as distressed prepayments. These results cannot be consistent with credit curing refinances {improving a credit history through time), because delinquency worsens not improves credit history. Our results therefore provide an alternative interpretation for the observed high rate of out-of-the-money prepayments of subprime loans, which is consistent with further credit deterioration.

Finally, the relationship between the extent or intensity of delinquency and default is non-linear. In fact, if a loan spends most of its life in delinquency, this actually implies a lower probability of default. These results are consistent with motivations such as free rent, income smoothing, and the value of delay.

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Notes

1. We also examine loans that do not terminate to account for all possible states.

2. Note that loans that arc in foreclosure proceedings have not fully terminated. In fact, a portion of these loans can be reinstated, prepaid, or modified (terms extended or other alterations made to reduce monthly payments), or experience other alternative outcomes. Researchers who examine these issues include Weagley [1988], Lawrence and Arshadi [1995], Ambrose and Capone [1996, 1998], Phillips and Rosenblatt [1997], Geppert and Karels [2001], Wang, Young, and Zhou [2002], and Lambrecht, Perraudin, and Satchell [2003].

3. In a short refinance, the lender forgives a portion of the debt and allows the borrower to restructure the delinquent mortgage into a new mortgage with a lower principal balance.

4. Industry reports have also examined the delinquency of mortgages. For example, Gjaja and Wang [2004] examine transition matrices of subprime loans for a single servicer.

5. Recall that default is defined as the beginning of foreclosure proceedings.

6. The probability of the third possible outcome, a mortgage continuing, equals one minus the probability of default minus the probability of prepayment.

7. As indicated in Greene [2000], multivariate probit allows the error terms to be correlated and thus relaxes the independence assumption of the multinomial logit. The assumption of a normal error term instead of logistic is also consistent with the first-stage error assumptions. In addition, in a J-dimensional problem J-1 probabilities must be considered. Therefore, in our case, with a three-dimensional problem, two probabilities must be considered.

8. In calculating cross-partial matrices:

$$E\left\{\left(\frac{\partial L_2}{\partial \theta_2}\right)\left(\frac{\partial L_2}{\partial \theta_1^T}\right)\right\}$$

and

$$E\left\{\left(\frac{\partial L_2}{\partial \theta_2^T}\right)\left(\frac{\partial L_1}{\partial \theta_1^T}\right)\right\}$$

where θ_1 and θ_1 are vectors of all estimated parameters, we account for the inclusion of the predicted intensity of delinquency variable, D_q, only.

9. The prepayment penalty indicator variable is included in the prepay specification only.

10. The impact of an increase in current LTV by one standard deviation elasticity on the probability of default is 316%. See Exhibit 5.

11. Lenders also can allow short sales (sales price < outstanding balance) to avoid the costs of foreclosure.

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Appendix

Exhibit 1: Dynamic Role of Delinquency

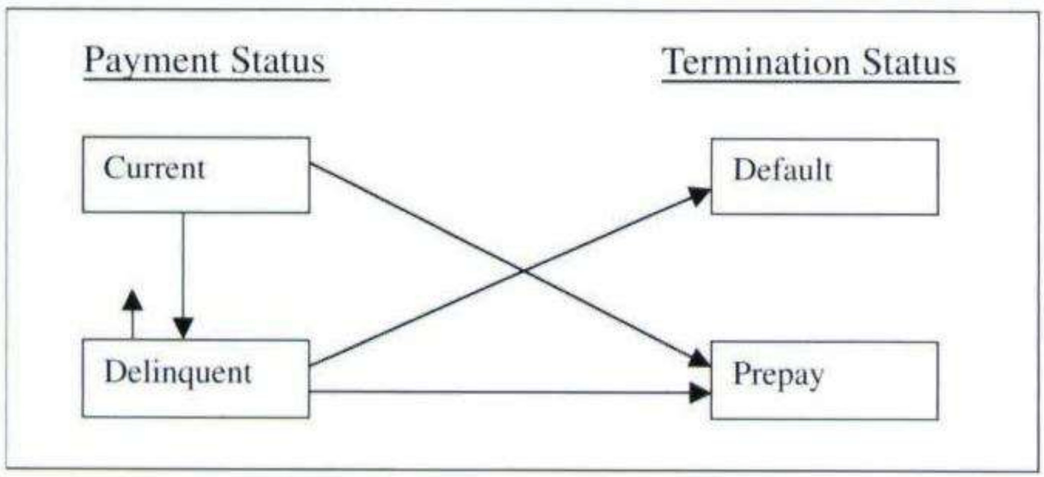

Exhibit 2: Description of Variables and Source

Variable	Source	Description
D_q	Loan-level data.	Provides the fraction of the observed life of the loan that it is delinquent—or the observed intensity of delinquency. For example, 0 indicates the loan is never delinquent, 0.5 that the loan is delinquent one-half of the time, and 1 that the loan is always delinquent (this is possible because some loans are seasoned before any information is available).
D_p	Loan-level data.	Indicates whether the loan is delinquent (= 1) or not (= 0).
d	Loan-level data.	Indicates whether the loan is defaulted (= 1) or not (= 0). A loan is defined as defaulted if it enters foreclosure or becomes real estate owned by the lender/investor.
p	Loan-level data.	Indicates whether the loan is prepaid (= 1) or not (= 0). Note that 1 – d – p = c, where c indicates whether the loan continues or is terminated. Loans are defined as prepaid when the loan is paid in full and the previous month's status was current or delinquent.
A	Loan-level data.	Provides the age of the loan expressed in months since the date of origination. Age^2, A^2, is also included in the estimation to capture any non-linear effects.
L	Loan-level data.	Origination loan-to-value ratio expressed in 100s so that 95 is a 95% loan-to-value ratio.
L_c	Office of Federal Housing and Enterprise Oversight and loan-level data.	Shows the current loan-to-value ratio derived from the balance on the loan and the updated value of the value of the property using the metropolitan area repeat sale price index. Also expressed in 100s.
F	Loan-level data.	Provides the credit score at origination reported for the loan.
U	U.S. Bureau of Labor Statistics.	Provides the metropolitan area reported unemployment rate for the previous month.
LD	Loan-level data.	Indicates that the loan has low or no documentation.
ΔI	Freddie Mac.	Provides the change in prevailing prime interest rates from the date of origination to the current date. The Primary Mortgage Market Survey is used.
P	Loan-level data.	Indicates whether a prepayment penalty is in effect for the current month. For example, for a loan with a prepayment penalty that lasts one year P = 1 if months < = 12 and P = 0 if months > 12.
S	Pool-level data.	Identifies 11 companies that provide the data to the repository (LoanPerformance.com). A dummy variable is constructed to capture any unique fixed effects associated with each data provider/servicer.

Exhibit 3: Summary Statistics for Estimation Data Set

	Mean	Std. Dev.	Minimum	Maximum
D_q	0.039	0.146	0	1
D_p	0.106	0.307	0	1
d	0.020	0.140	0	1
p	0.041	0.198	0	1
A	14.825	13.871	1	95
L	90.973	14.049	20	125
L_c	83.612	15.327	11	125
F	660.188	71.600	373	827
U	5.105	2.088	1.2	19
LD	0.294	0.455	0	1
ΔI	-0.501	0.743	-3	2
P	0.379	0.485	0	1
Absc	0.028	0.164	0	1
Cbass	0.026	0.159	0	1
Centex	0.030	0.171	0	1
Dlj	0.078	0.268	0	1
Equicredit	0.064	0.245	0	1
Icifc	0.039	0.195	0	1
Independent	0.026	0.159	0	1
Residential Funding Corporation	0.440	0.496	0	1
Ryland	0.190	0.392	0	1
Sasco	0.079	0.269	0	1
Number of observations	22,799			

D_q is the intensity of delinquency. D_p indicates when the loan is delinquent. d indicates the loan has defaulted. p indicates the loan has prepaid. A is age. L is the origination loan-to-value ratio. L_c is the current loan-to-value ratio. F is the FICO score. U is last month's unemployment rate. LD is a low or no documentation loan. ΔI is the cumulative change in interest rates since origination. P is the prepay penalty if in force for the current month. The other variables are dummy variables for each data provider.

Exhibit 4: Double-Hurdle Results

	Probability of Delinquency (Dp)		Intensity of Delinquency (Dq)	
	coeff	z	coeff	z
A	1.781	25.8	-0.188	-6.1
A^2	-1.009	-19.6	0.121	5.1
L	-0.238	-4.2	-0.073	-3.4
L_c	0.382	6.2	0.074	3.2
F	-0.356	-13.6	-0.116	-9.1
U	-0.023	-1.0	0.008	0.7
LD	0.064	2.6	-0.001	-0.1
Absc	-0.002	-0.1	0.026	2.9
Cbass	-0.033	-2.3	0.071	11.1
Centex	0.033	1.8	-0.012	-1.6
Dlj	0.108	4.8	-0.004	-0.4
Equicredit	-0.266	-13.0	0.085	8.3
Icifc	0.014	0.6	0.019	1.8
Independent	0.035	1.6	-0.002	-0.3
Ryland	0.023	1.0	0.019	1.8
Sasco	-0.162	-5.8	0.081	4.6
Constant	-1.217	-42.5	0.106	5.0
Sigma			0.385	52.0

All variables are transformed so that the mean is zero and the standard deviation is 1.

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Exhibit 5: Seemingly Unrelated Bivariate Probit Results

	Probability of Default (π_d)			Probability of Prepay (π_p)		
	Coeff	Z-stat	Murphy Topel Z-stat	Coeff	Z-stat	Murphy Topel Z-stat
D_q	0.179	2.12	2.09	0.339	4.71	4.68
$(D_q)^2$	-0.054	-1.38	-1.37	-0.122	-2.79	-2.79
D_p	-0.232	-2.36	-2.33	-0.395	-5.87	-5.82
A	1.252	9.01	8.95	0.540	6.61	6.55
A^2	-0.780	-8.60	-8.53	-0.414	-7.73	-7.70
L	-0.508	-6.43	-6.42	0.161	3.80	3.78
L_c	0.593	7.27	7.26	-0.207	-4.86	-4.79
F	-0.273	-8.28	-8.24	0.073	3.43	3.42
U	-0.090	-2.30	-2.25	-0.066	-3.31	-3.31
ΔI	-0.060	-2.68	-2.68	-0.048	-2.60	-2.60
P				-0.144	-8.15	-8.15
Cbass	-0.008	-0.33	-0.32	-0.032	-1.49	-1.49
Centex	-0.026	-1.29	-1.29	0.013	0.72	0.72
Dlj	0.021	0.81	0.80	0.066	4.28	4.27
Equicredit	-0.012	-0.42	-0.39	-0.069	-3.03	-3.02
Icifc	0.044	2.12	2.12	0.048	3.25	3.26
Independent	0.065	3.71	3.71	0.036	2.58	2.54
Ryland	0.021	0.91	0.91	0.074	4.38	4.28
Sasco	-0.026	-0.76	-0.75	-0.035	-1.98	-1.97
Constant	-2.341	-75.74	-75.74	-1.804	-109.24	-109.24
Rho	-0.709	-1.00	-0.30			

All variables, including the dummy variables, are transformed so that the mean is zero and the standard deviation is 1. The excluded data provider is the Residential Funding Corporation, which includes both RFC Home Equity and RFC Master.

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Exhibit 6: Direct and Indirect Effects of FICO on Default and Prepayment Probabilities

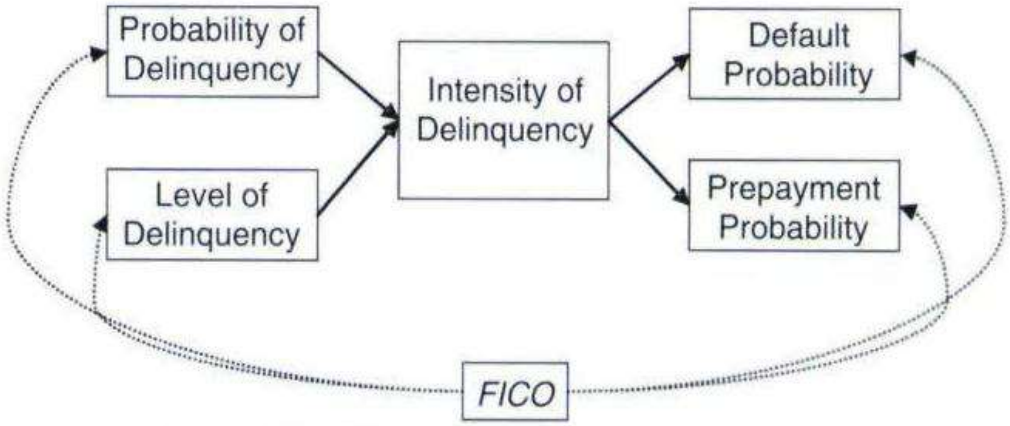

Exhibit 7: Effect of Predicted Intensity of Delinquency on Termination

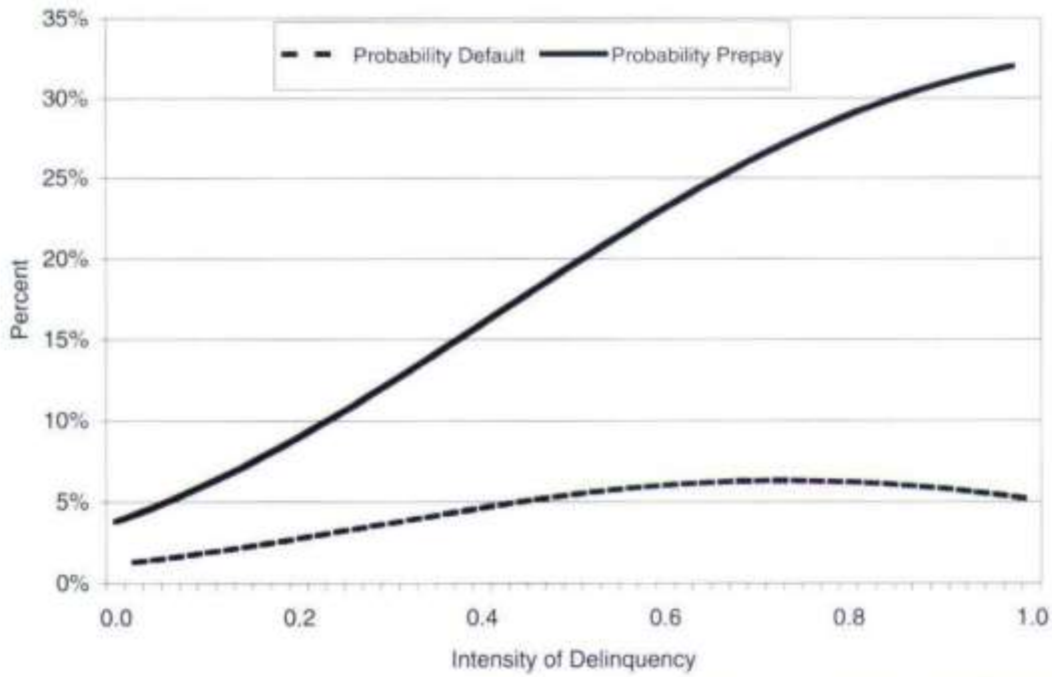

Exhibit 8A: Effect of LTV at Origination on First- and Second-Stage Estimates

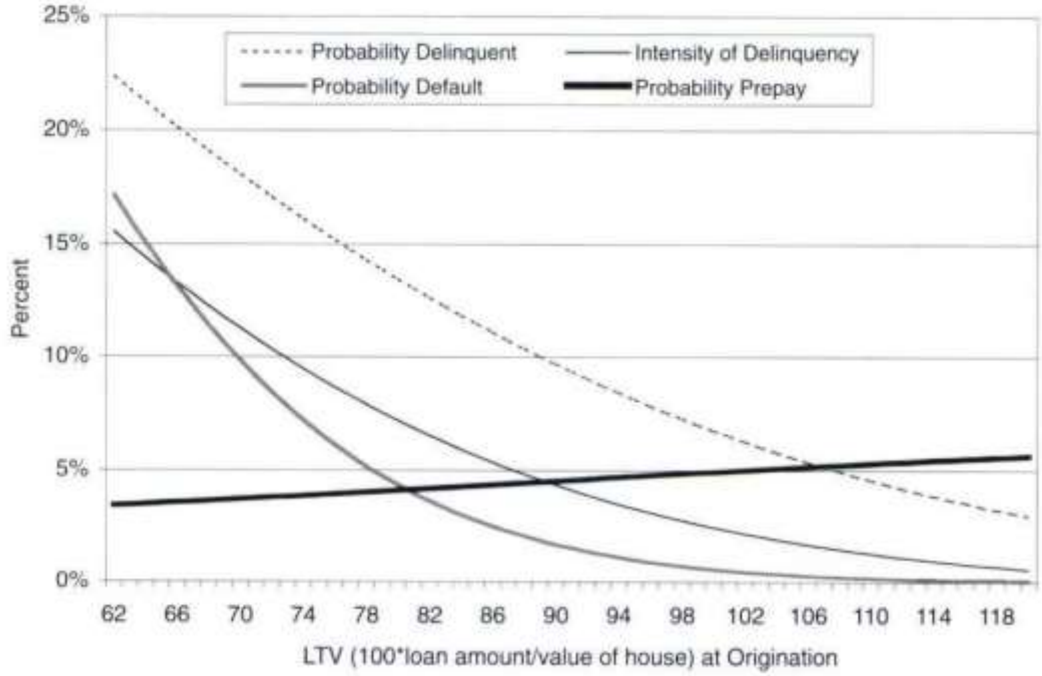

Exhibit 8B: Effect of Current LTV on First- and Second-Stage Estimates

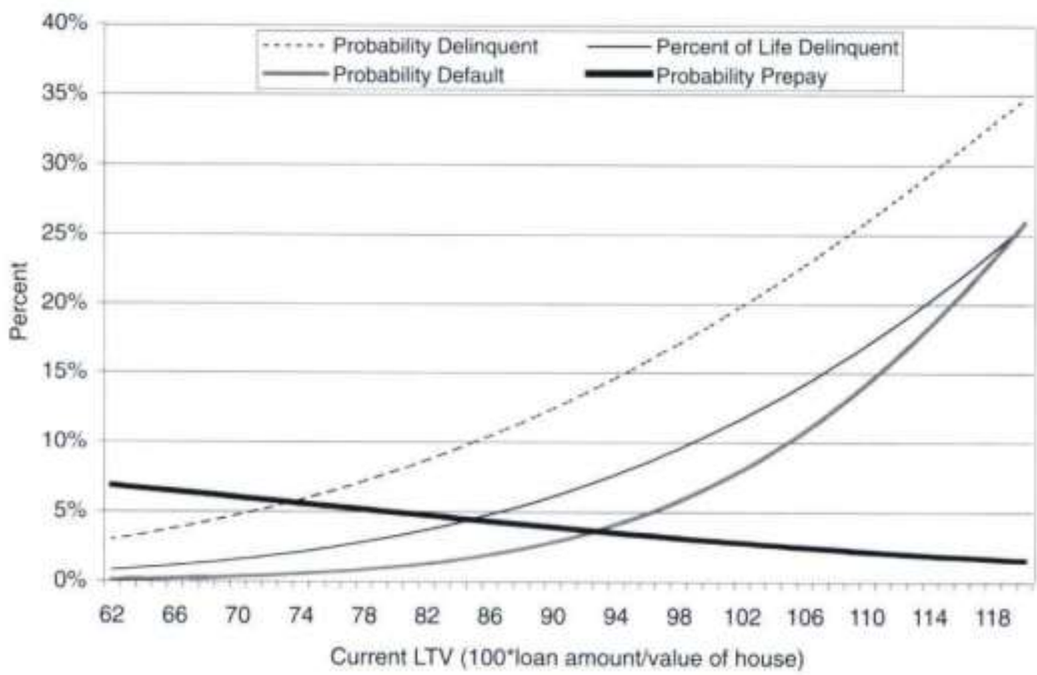

Exhibit 9A: Predicted Probability of Prepayment for Various Current Equity Positions and Intensity of Delinquency

		Intensity of Delinquency Rate	
		Low	High
Current LTV*	Low	0.031	0.080
	High	0.023	0.063

Exhibit 9B: Predicted Probability of Default for Various Current Equity Positions and Intensity of Delinquency

		Intensity of Delinquency Rate	
		Low	High
Current LTV*	Low	0.001	0.004
	High	0.043	0.091

* Direct effect only. Low and high are defined as a one standard deviation above and below the mean.

Exhibit 10: Effect of FICO on Delinquency

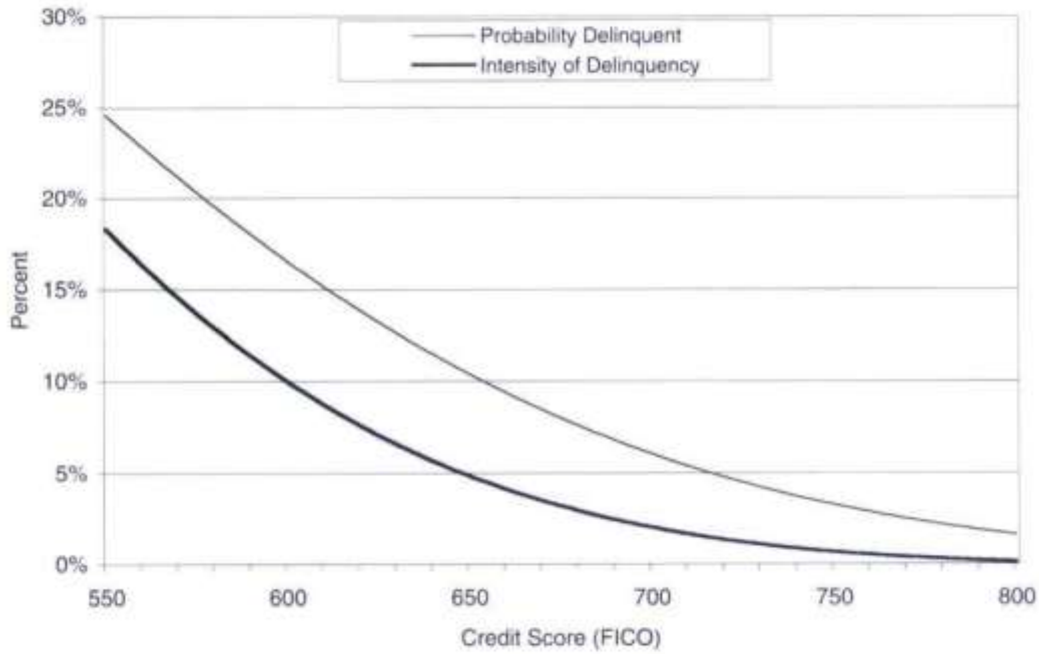

Exhibit 11: Effect of FICO on Termination

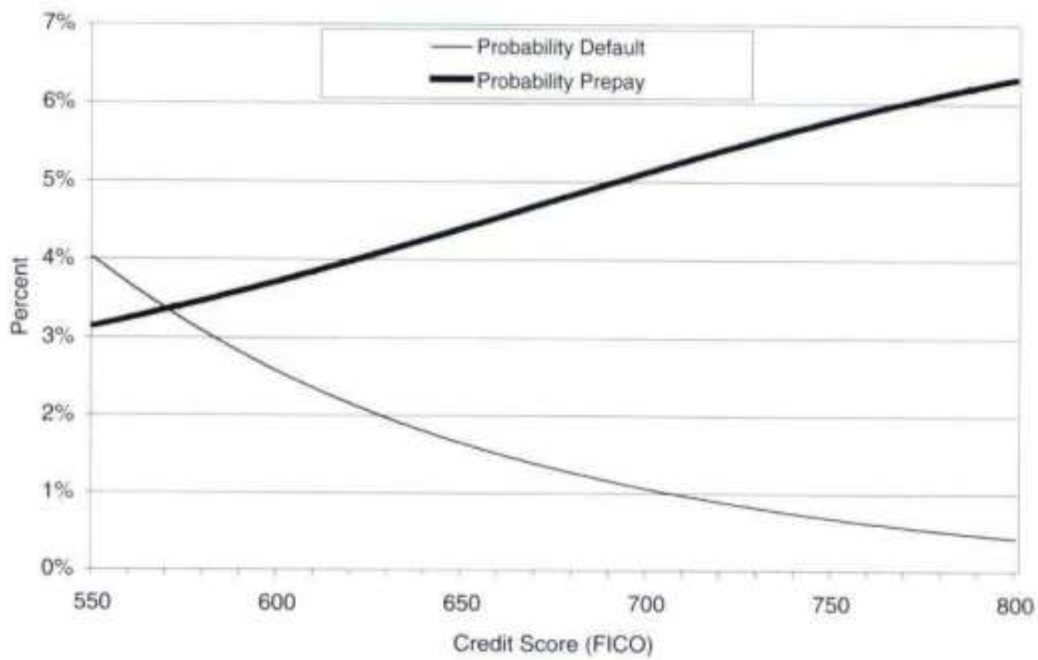

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Exhibit 12: One Standard Deviation Elasticity

Variable	Probability Delinquent	Intensity of Delinquency (Percent of Life Delinquent)	Probability Default	Probability Prepay
F	-56%	-75%	-47%	22%
L	-41%	-57%	-80%	13%
L_c	90%	144%	316%	-33%
A	170%	66%	222%	-22%
U	-2%	1%	-13%	-5%
˜I			-15%	-9%

Exhibit 13: Effect of Change in Interest Rates on Termination

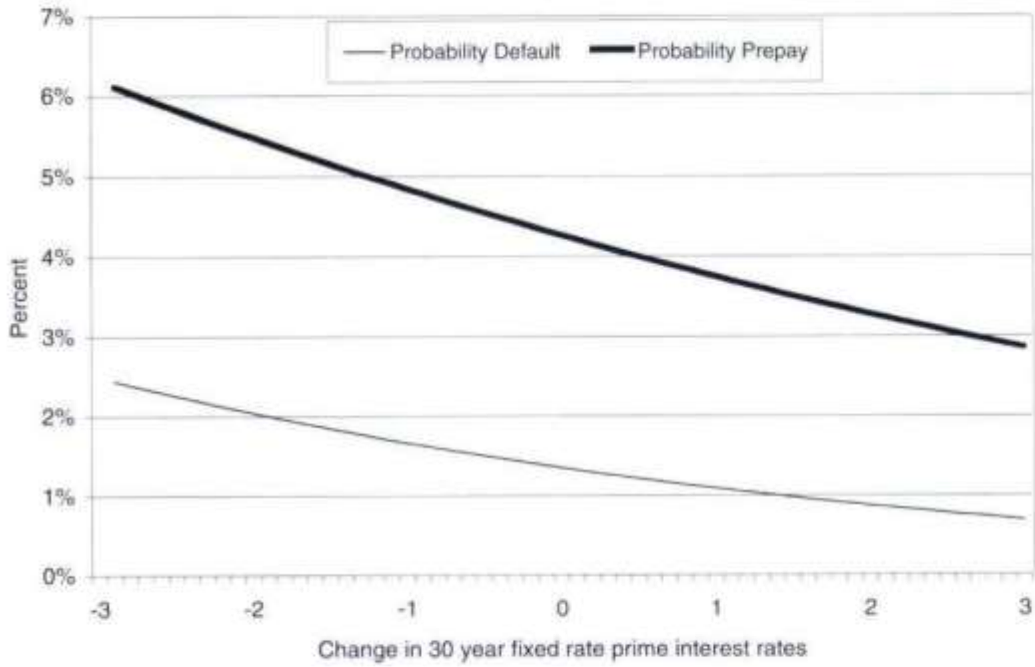

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Exhibit 14: Fixed and Discontinuous Effects—Percent Change

Variable	Probability Delinquent	Intensity of Delinquency (Percent of Life Delinquent)	Probability Default	Probability Prepay
LD	22%	20%	-4%	-9%
P				-49%
Absc	29%	136%	10%	2%
Cbass	24%	324%	36%	14%
Dentex	14%	-25%	-33%	9%
Dlj	66%	54%	6%	30%
Equicredit	-81%	-44%	5%	-15%
Icifc	33%	101%	78%	66%
Independent	32%	22%	154%	41%
Ryland	20%	50%	14%	44%
Sasco	-46%	41%	-1%	16%

